Roll No. .......................... **Total Pages : 03**

# BT-4/M-20 **34091**

# MATHEMATICS-III

# AS-201N

Time : Three Hours] [Maximum Marks : 75

**Note :** Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

## Unit I

**1.** (a) Obtain a Fourier series to represent $e^{-ax}$ from $x = -\pi$ to $x = \pi$. **7½**

(b) Obtain the half-range sine series for $e^x$ in $0 < x < 1$. **7½**

**2.** (a) Find the Fourier cosine transform of $e^{-x^2}$. **7½**

(b) State and prove convolution theorem for Fourier Transforms. **7½**

## Unit II

**3.** (a) If $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$; then prove that :

(i) $\tanh\frac{u}{2} = \tan\frac{\theta}{2}$

(ii) $\cosh u = \sec\theta \cdot$ 7½

(b) Separate $\sin^{-1}(\cos\theta + i\sin\theta)$ into real and imaginary parts, where $\theta$ is a positive acute angle. 7½

**4.** (a) Show that the function $f(z) = \sqrt{|xy|}$, is not analytic at the origin, even though Cauchy-Riemann equations are satisfied thereat. 7½

(b) If $f(z)$ is a regular function of $z$, prove that : 7½

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$

## Unit III

**5.** (a) The contents of three urns are 1 white, 2 red, 3 green balls; 2 white, 1 red, 1 green balls and 4 white, 5 red, 3 green balls. Two balls are drawn from an urn chosen at random. These are found to be one white and one green. Find the probability that the balls so drawn came from the third urn. 7½

(b) Fit a binomial distribution to the following frequency distribution : 7½

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x$ | : | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $f$ | : | 13 | 25 | 52 | 58 | 32 | 16 | 4 |

6. (a) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D. of the distribution. **7½**

(b) A random variable $x$ has the following probability function :

| $x$ | : | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | : | $0.1$ | $k$ | $0.2$ | $2k$ | $0.3$ | $k$ |

Find $k$, mean, variance and S.D. **7½**

## Unit IV

**8.** Using simplex method, solve the following linear programming problem :

Minimize $Z = x_1 - 3x_2 + 2x_3$

Subject to constraints

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0.$$

**15**

**9.** Using dual simplex method, solve the following LPP :

Maximize $Z = -3x_1 - x_2$

Subject to

$$x_1 + x_2 \geq 1$$
$$2x_1 + 3x_2 \geq 2$$
$$x_1, x_2 \geq 0.$$

**15**