

Roll No.

Total Pages : 04

BT-4/M-20

34114

MATHEMATICS-III

AS-201N (Opt. I)

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Find the Fourier series of the function given by : $7\frac{1}{2}$

$$f(x) = \begin{cases} 0 & -\pi \leq x < 0 \\ \pi & 0 \leq x < \pi \end{cases}$$

- (b) Develop $\sin\left(\frac{\pi x}{l}\right)$ in half-range cosine series in the range $0 < x < l$. $7\frac{1}{2}$

2. (a) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$. $7\frac{1}{2}$

- (b) Using Parseval's identity, prove that : $7\frac{1}{2}$

$$\int_0^{\infty} \frac{\sin 3t}{t(9+t^2)} .dt = \frac{\pi}{18} (1 - e^{-9})$$

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Unit II

3. Use the simple method to solve the following LP problem :

$$\text{Maximize } z = 3x_1 + 5x_2 + 4x_3$$

subject to :

$$2x_1 + 3x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

$$x_1, x_2, x_3 \geq 0. \quad \mathbf{15}$$

4. (a) Using Graphical method : **7½**

$$\text{Maximize } z = -3x_1 - x_2$$

subject to :

$$x_1 + x_2 \geq 1$$

$$2x_1 + 3x_2 \geq 2$$

$$x_1, x_2 \geq 0$$

- (b) Explain the following terms : **2½×3**

(i) Feasible Solution

(ii) Convex Region

(iii) Unbounded Solutions.

Unit III

5. (a) If $u = \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$, then prove that : $7\frac{1}{2}$

(i) $\tanh \frac{u}{2} = \tan \frac{\theta}{2}$

(ii) $\cosh u = \sec \theta$

(b) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic.
Find a function v such that $f(z) = u + iv$ is analytic.
Also express $f(z)$ in terms of z . $7\frac{1}{2}$

6. (a) Evaluate : $7\frac{1}{2}$

$$\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz,$$

where C is the circle $|z|=3$.

(b) Evaluate :

$$\int_C (y - x - 3x^2 i) dz$$

where C is the straight line from $z = 0$ to
 $z = 1 + i$. $7\frac{1}{2}$

Unit IV

7. (a) Three urns contain 6 red, 4 black; 4 red, 6 black
and 5 red, 5 black balls respectively. One of the

urns is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the first urn. $7\frac{1}{2}$

(b) In a normal distribution, 35% of the items are under 40 and 10% are over 60. Find the mean and standard deviation of the distribution. $7\frac{1}{2}$

8. (a) A random variable X has the following probability distribution :

$$x \quad : \quad -3 \quad -2 \quad -1 \quad 0 \quad 1$$

$$P(x) \quad : \quad 0.2 \quad k \quad 0.3 \quad 3k \quad 0.1$$

Find the value of k and calculate mean and variance.

$7\frac{1}{2}$

(b) In 800 families with 5 children each, how many families would be expected to have (i) 2 boys and 3 girls (ii) at the most two girls ? (Assume probabilities for boys and girls to be equal). $7\frac{1}{2}$